# In Memoriam Michael (Miki) Neumann (1946-2011)

by Michael Tsatsomeros<sup>1</sup>



With a quiet smile, a sense of self-effacement, some self-deprecation, and always with a grain of humor, Miki liked to claim that

# "... all I know is how to multiply matrices."

This statement epitomized Miki's philosophy. You build knowledge methodically, upon fundamentals, with new ideas and careful reasoning. In all, you stay humble, measured, appreciative of your opportunities and confident of your accomplishments.

Miki was a family man with paramount loyalty to his roots and faith. Married for nearly 38 years, Miki and Helen raised two wonderful children, Joseph and Rachel. It only took a minute listening to Miki talk about them to understand how loving and proud he was of his children. At his induction as the Stuart and Joan Sidney Professor of Mathematics, Miki described how his father introduced him to calculus using the book *Calculus Made Easy*. When he became a father and was absorbed in his research projects, he recalled that his young daughter Rachel told friends, "My daddy is not a man, he's a mathematician."

# MICHAEL NEUMANN'S CAREER

Michael (Miki) Neumann, was born in Jerusalem in 1946. He earned his B.Sc. from Tel Aviv University in 1970 and his Ph.D. in Computer Science from London University in 1972. Miki taught at the University of Reading (72-73), the Technion-Israel Institute of Technology in Haifa (73-75), the University of Nottingham (75-80), and the University of South Carolina (80-85). In 1985 he came to the University of Connecticut as Professor of Mathematics. Miki served as Head of the Department of Mathematics from 2003 until the time of his death. He was named a Board of Trustees Distinguished Professor in 2007, and appointed as

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the Stuart and Joan Sidney Professor of Mathematics in 2010. In 2007 Miki was elected to the Connecticut Academy of Arts and Sciences.

Miki's research received much recognition. His work is remarkable not only for its sheer quantity but even more so for its range and depth. As testament for his influence to Linear Algebra and the respect for his professional judgement, we need only note the numerous citations of his work, the consistent funding he received, as well as his service on the editorial boards of Linear Algebra and its Applications, the Electronic Journal of Linear Algebra, and Operators and Matrices.

Miki was also a dedicated teacher. In the classroom, Miki's priority was to communicate knowledge and to inspire his students to learn. He was the kind of teacher one appreciates more with time, because his teaching aspirations were more about substance and less about fanfare.

As the years go by, his former graduate students become more and more convinced that Miki was a model adviser and mentor. By his example, he impressed in them the value of dedication to the profession, the value of academic integrity, and the importance of service. He taught his students masterfully the technical knowledge needed to start and sustain a research career, and also taught them the value of being patient and persevering in the quest for solutions.

Miki remained very supportive of his students, colleagues and friends throughout their careers, and offered much needed help to many of us through personally trying times. He was genuinely interested in the well-being of his academic progeny beyond their professional lives.

In his role as an administrator and in decision making positions, Miki prided himself, sometimes even obsessively, on being fair, respectful and appreciative of his colleagues and their efforts.

To conclude, Miki will be remembered as the embodiment of a loving parent, a great mathematician and a superb human being. In the words of his colleague Stuart Sidney, we will miss the look of joyful expectation on his face, as if the next moment would hold a wonderful surprise.

## MICHAEL NEUMANN'S RESEARCH

Miki viewed his research as divided into five main and interconnected areas of Linear Algebra. They are described below, along with his contributions, largely in his own words.

#### 1. Iterative Methods for Linear Systems of Equations

Miki made an important contribution to the subject by giving an exact characterization of four major subspaces that are important in determining, for a system of linear equations Ax = b and an iterative scheme  $x_i = Tx_{i-1} + c$ , the exact or approximate solution to which the iterates might converge (see [1, 3, 4]). Two of the subspaces are the nullspace and range of A. The other two depend on the splitting of A into A = M - N and the iteration matrix T which they induce. Prior to the paper [3] by Berman and Neumann, knowledge of these subspaces was mostly confined to a narrow class of least-squares problems. Miki's work on the convergence of iterative methods for singular systems led to his interest in the topic of nonnegative matrices because, in the presence of a nonnegative iteration matrix, much can be deduced about the necessary and the sufficient conditions for the convergence of an iteration scheme from both the numerical and the graph theoretical structure of the matrix (see [5, 7, 8, 15, 46]). Miki's work on iterative methods deals also with conditions for convergence in the presence of iteration matrices which are not nonnegative, but have other recurring properties such as paracontracting, (see [43]), cyclicity (see [20, 38, 45, 53]), and positive definiteness (see [42]). Paracontracting matrices turned out to be particularly useful in parallel methods for solving systems of equations (see [43, 51, 57, 61, 62, 98]). Miki received research grants over a period of several years for his work on parallel methods.

#### 2. Nonnegative Matrix Theory

At the time Miki became interested in the applications of nonnegative matrices to iterative methods

for solving linear systems Ax = b, Richard Varga had produced a beautiful theory for the convergence of iterative methods induced by regular splittings when A is nonsingular. The transition to singular systems necessitated understanding the deeper relation of a nonnegative matrix to its Perron eigenspace. It is here that Miki made use of the existence of a nonnegative basis for the (generalized) Perron eigenspace of a nonnegative matrix. The proof of the existence of such a basis, due to Rothblum and Richman-Schneider, had a substantial combinatorial aspect to it. Yet, its use for investigating the convergence of iterative methods in [7] required no assumptions about the combinatorial structure of the splittings or the resulting iteration matrix. This made Miki and others wonder for many years whether there was a completely analytic proof of the existence of a nonnegative basis, until it was found in 1988 in joint work (see [52]) with Robert Hartwig and Nicholas Rose. The proof rested on the asymptotic expansion of the resolvent operator. Miki continued to work, mainly with Hans Schneider, on the relation between the analytic and combinatorial structure of nonnegative bases of nonnegative matrices and on how to generate them (see [60, 70]). Another aspect of Miki's work on nonnegative matrices dealt with questions on convexity and concavity of the Perron root of a nonnegative matrix. Actually, these questions fall in the area of perturbation problems for nonnegative matrices (see [28, 35, 102, 116, 128, 144, 145, 146]). The main tools used here were generalized inverses of singular and irreducible M-matrices, on which Miki wrote over 20 papers (see e.g., [27, 28, 35, 74, 76, 78, 83]). One of the papers, written jointly with Steve Kirkland, studies the form of the growth of the population in a Leslie population growth model (see [71]). Quite surprising conclusions concerning the effect on the rate growth of the population due to change in fecundity at different ages in the population are found there.

Computational and perturbation questions for stochastic matrices have been considered in quite a few of Miki's papers. For example, a well known idea of Carl Meyer is the decoupling of a Markov chain - or more practically its transition matrix - using Perron complementation. The task is to compute (possibly in parallel) by a divide and conquer method its stationary distribution vector. Miki and his co-authors showed that it is possible to extend this approach to the computation of the mean first passage times matrix (see [99, 111]). These papers demonstrate that using this approach is very economical in terms of the count of arithmetic operations. Other papers of Neumann on Markov chains are mostly concerned with estimating the error as a result of the computation (see [88, 124, 126, 141, 148]). Some of these papers are also concerned with special Markov processes that occur, for example, in small-world problems (see [134]). Perron complementation has appeared in many other papers by Miki and on a variety of topics. For example, on the inverse M-matrix problem (see [104], on totally nonnegative matrices (see [110]), on upper-estimates of permanents of doubly stochastic matrices (see [125]), and in the context of Soules bases (see [140]).

The topic of Soules matrices mentioned above can be seen as part of Miki's work on M-matrices and on the inverse M-matrix problem. Miki's work on the inverse M-matrix problem starts with his joint paper with Lewin [13] in which the (0, 1)-matrices that are inverse M-matrices are characterized in a completely combinatorial way. This is among the first papers on the inverse M-matrix problem. Other papers on this topic are [55, 75, 82, 89, 96, 97, 104, 107, 138, 140, 143]. In this list, papers [89, 107, 138, 140, 143] concern the fascinating topic of the Soules matrices. Once it was realized that Soules matrices are inverse M-matrix theorists became possible: on the topic of MMA-matrices (Friedland, Hershkowitz, Schneider, and Stuart), on ultrametric matrices (McDonald, Nabben, Schneider, Tsatsomeros, Varga), among others. Soules matrices and their eigenbases have also helped in the understanding of the inverse eigenvalue problem for nonnegative matrices (see [107, 138, 143]).

The work on the Soules bases had recently become useful in a new area of interest for Miki, namely, recognizing objects and reconstructing images through nonnegative matrix factorizations (see [129]). Such factorizations are different from other methods of image reconstruction, because they make use of nonnegative constraints that allow additive approximations without cancellation. As a consequence, features of an image emerge with a sharper focus when reconstructed. An efficient algorithm for solving nonnegative matrix factorization problems is proposed in [152].

# 3. Nonnegative Dynamic Systems

In biological systems, such as models of predator-prey relationships, one can ask the following question: what is the set of all symbiosis points? These are initial states of evolving populations from which the populations are destined to become and remain non-decreasing. This concept, due to Neumann and Tsatsomeros ([59]) is a special case of the so-called cone reachability problem. This problem entails finding the set of all points  $x_0 \in \mathbb{R}^n$  such that the trajectory emanating from them enters the nonnegative orthant at some time  $t = t(x_0)$  and remains in the orthant for all time thereafter. Miki and a colleague from Montreal, Ron Stern, investigated this problem and developed an approach to its solution based on an eigenspace deflation process (see [31, 33, 37], the latter paper also with A. Berman).

Predator-prey problems can often be cast as systems of linear differential equations. Suppose now that we try to solve such a system of differential equations by means of a numerical approximation method, e.g., Euler's scheme. Then one question to be answered is this: If the continuous trajectory emanating from an initial state becomes and remains nonnegative, then does a discrete trajectory, (e.g., one generated by Euler's scheme) also become and remain nonnegative? The answer to this question involves some delicate issues, particularly about points that reach and stay on the boundary of the nonnegative orthant. Using both asymptotic expansions and much eigenspace analysis, Miki, with Stern and Tsatsomeros, showed that under certain mild restrictions on the size of the time steps, the continuous and the discrete reachability cones coincide. This quite surprising, qualitative result was at the heart of the work of Miki's second Ph.D. student, Michael Tsatsomeros. Miki, with Stern and Berman, published a book on reachability cones and related topics entitled *Nonnegative Matrices in Dynamic Systems*.

#### 4. Parallel Methods for Solving Linear Systems

In the mid-80s, papers began to appear in which parallel computing methods (synchronous iteration) for solving large linear systems of equations were suggested. In some of these methods, the different operators to be successively applied were nonnegative matrices. At first, Miki tried to build a framework for analyzing parallel methods. Miki then went on to investigate chaotic, i.e., asynchronous iteration methods. The first task was to try and express the procedure in some mathematical fashion. Once this was done, then a concept mentioned earlier, that of paracontracting operators, helped in proving convergence results (see [43, 47, 51, 57, 61, 62, 72]]). In one interesting paper (see [57]) it is proved that when increasing parallelization is applied to certain problems, then at first there is an improvement in the speed-up rate of the computation, but beyond a certain point the rate levels off because of communication overheads.

#### 5. The Algebraic Connectivity of Graphs

There are types of matrices (e.g., adjacency matrices and Laplacian matrices) that can be used to model graphs. Certain parameters associated with these matrices can reveal properties of a graph, for example create a measure for the degree of its connectedness. One of these parameters is the second smallest eigenvalue of the Laplacian matrix, known as the algebraic connectivity of a graph. The computation of this eigenvalue can be expensive and therefore it becomes necessary to approximate it well. In the last several years, Miki had been particularly engaged in developing sensitive, yet economical, estimators for this quantity. Such estimates can be derived from the generalized inverse of the Laplacian matrix. At the heart of this approximation is the idea of the inverse eigenvalue iteration that can be extended to the nonzero spectrum of the Laplacian matrix. Much of this work is concerned with the equality case between the algebraic connectivity and its upper estimates, leading to interesting extremal graphs. Miki worked on problems associated with the algebraic connectivity of a graph with quite a few people: Barik, Kirkland, Molitierno, Pati, and Shader producing papers [80, 85, 86, 94, 103, 109, 112, 113, 119, 122, 123, 137, 142, 148].

## MICHAEL NEUMANN'S DOCTORATE STUDENTS

Miki advised nine Ph.D. students, listed below, all of whom went on to distinguished careers in Mathematics. What is more, they were all like family to Miki and Helen.

Valerie Miller (1985); Michael J. Tsatsomeros (1990); Mei Gao (1993); Yonghong Chen (1994); Lixing Han

(2000); Jason J. Molitierno (2001); Jianhong Xu (2003); Minerva Catral (2005); Upendra Prasad (2009).

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